1 Unit:-3:

Partial Order Set or (Poset):

A non-empty set A, together with a binary relation R is said to be a poset if R is partial order relation.

Reflexive
anti symmetric
transitive

Poset is denoted by (A, R) or $(A, \leq)$

Exp:- The set of S many collection of sets. The relation $\subseteq$ read as "is subset of" is partial order of S. $(S, \leq)$

Proof:-

① ~~Ref~~Reflexive:-

Since $A \subseteq A$ for any subset of S.

② Antisymmetric:-

If $A \subseteq B$ and $B \subseteq A$ $\forall A, B \in S$ then $A = B$

③ Transistive:-

if $A \subseteq B$ and $B \subseteq C$ for any set $A, B, C \in S$ then $A \subseteq C$.

Hence $(S, \leq)$ is Poset.

Total order Relation or Linear Order Relation:

A Relation R defined on a set A; is said to be a total order Relation in A if

(i) R is partial order Rel^n in A

(ii) Every pair of elements of A is comparable ~~with~~ WRT to R.

(Law of Dichotomy)

Exp:- set of natural number, Relation is defined by " x is less than or equal to y" is ~~t~~ tot order relⁿ or not.

Exp:- R be relation in the set A = {1, 2, 3, 4, 5, 6} is ~~total~~ total order relⁿ or not. R = Divides.

Exp:- Set of Integer, R = Division
No be (2) / (-2) & (-2) / 2 but 2 ≠ -2 ~~total~~ antisy

~~(1,1) (2,2) (3,3) (4,4) (5,5) (6,6)~~

Exp:- Set of Positive Integer

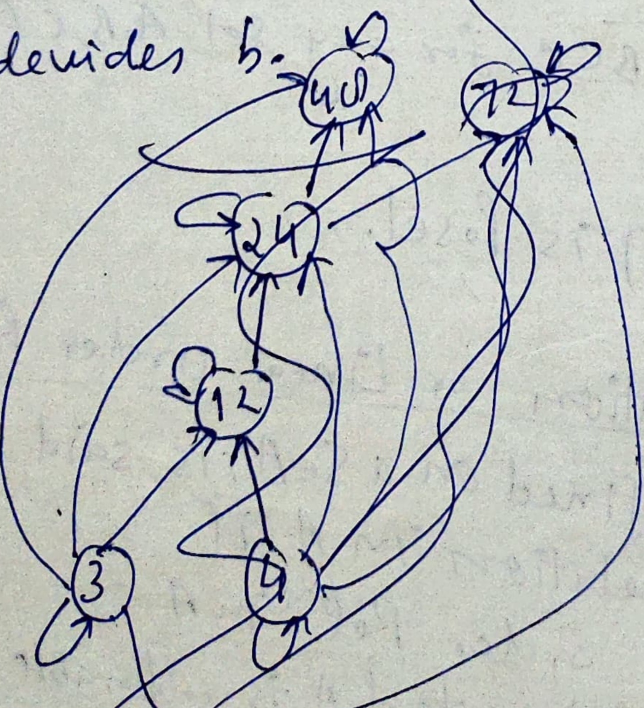
Exp:- Let X = {a, b, c}, (P(X), ⊆) is totalorder set or not.

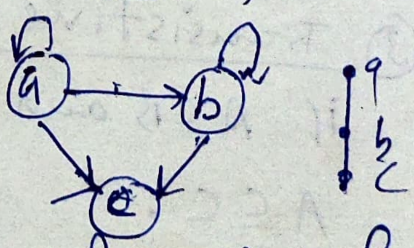
A = {a, b, c}
<=
Rel (a,a) (a,b) (b,c) (a,c) (b,b) (c,c)

Digraph of Poset:

Prob:- (A, ≤) wher A = {3, 4, 12, 24, 48, 72}

and relation ≤ be such that a ≤ b if a devides b.





① Remove cycle

⇒ ② Remove transitive edges.

③ Remove arrow upword direction

④ circle by .

②

1 Hasse diagram:-

① $(A, \leq)$, $A = \{3, 4, 12, 24, 48, 72\}$,
$\leq$ define such that $a \leq b$ if a divides b.

② $B = \{2, 3, 4, 6, 12, 36, 48\}$, 'divides'

③ A be set of factors of positive integer 30,
Relation "divides"

④ $X = \{a, b, c\}$, $(P(X), \subseteq)$. Hasse dia-

Upper bound:-

Let $(A; \leq)$ be a Poset $a, b \in A$ then an elem
$c \in A$ is upper bound of a, b if
$a \leq c$ and $b \leq c$

Least upper bound (supremum) (V)

l is least upper bound of a, b.

① if $a \leq l$ and $b \leq l$

for every element $l' \in A$

② if $a \leq l'$ and $b \leq l'$ and $l \leq l'$

Lower bound of a and b

Lower bound:

if $d \leq a$ and $d \leq b$.

Greatest lower bound (Infimum) $\wedge$

(i) $d \leq a$ and $d \leq b$

(ii) for some d' if $d' \leq a$ and $d' \leq b$

and $d' \leq d$

Maximal element:

A element 'a' in the Poset $(A, \leq)$ is said to be maximal if there is no element $b \in A$ such that $a < b$.

Minimal element: An element 'a' in Poset $(A, \leq)$ is said to be minimal if there is no element $b \in A$ such that $b < a$.

Greatest element (Last element or Unit element):

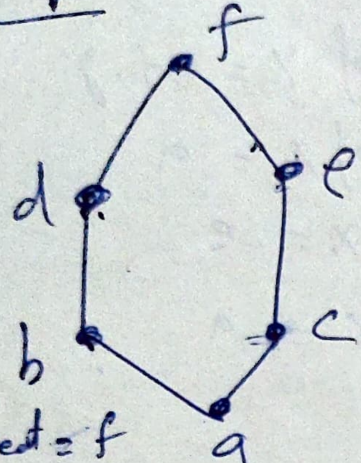
a is said to be greatest element of A if $x \leq a$ for all $x \in A$.

Least (First or Zero) element:

a is said to be least of A if $a \leq x$ for all $x \in A$.

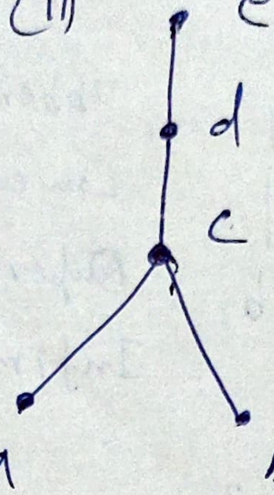
Exp:-

(i)



great = f
le = a

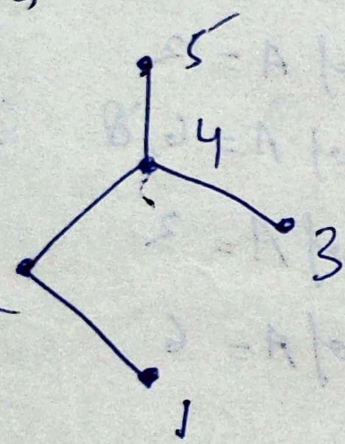
(ii)



Maximal = e
minimal = a, b

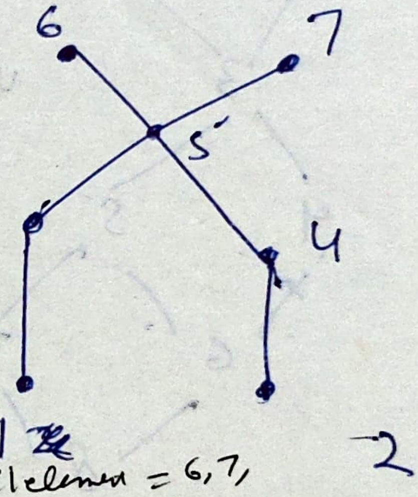
great = e
least = None

(iii)



greatest = 5
least = None

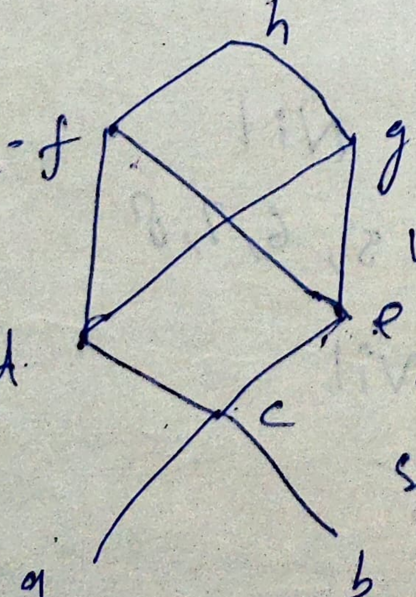
(iv)



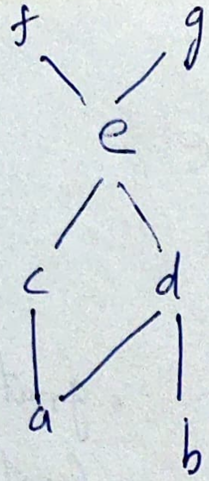
Maximal element = 6, 7,
Minimal element = 1, 2

greatest = None
least = None

Problem:-



	a, b	c, d, e	f, g
Upper	c, d, e, f g, h	f, g, h	h
Lower	Nil	c, a, b	a, b, c, d, e
Sup	c	Nil	h
Infi	Nil	c	Nil

① 

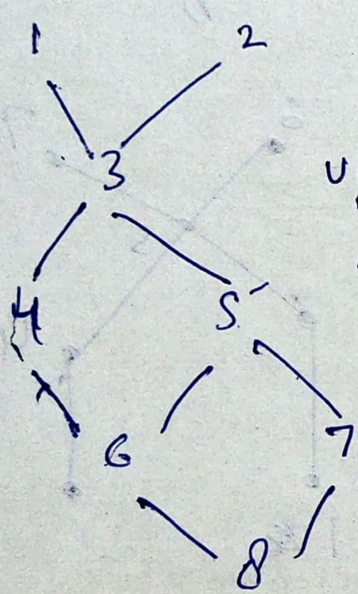
X = e, c d

Ubber bound of X = e, f, g

Lower bound of X = a

Superemum of X = e

Infimumn of X = a

② 

A = {2, 3, 6}

Upper bound of A = 2

Lower bound of A = 6, 8

Supremum of A = 2

Infimum of A = 6

③ Repeat the above problem for Subset B = {1, 2, 5}

Ubber bound of B = Nil

Lower bound of B = 5, 6, 7, 8

Supremum of B = Nil

Infimum of B = 5

Latice:

A Poset $(L, \leq)$ is said to be Lattice if every two element in the set L has a unique least upper bound (sup) & a unique greatest lower bound (inf).

or

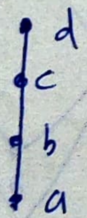
The Poset $(L, \leq)$ is a lattice if for every $a, b \in L$ sup(a,b) and inf(a,b) exists in L.

$\sup(a,b) = a \vee b$ = a joint b

$\inf(a,b) = a \wedge b$ = a meet b

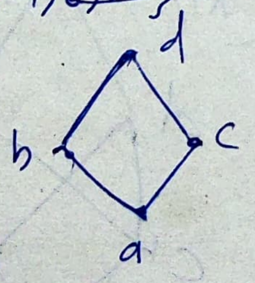
Exp:- ~~A = {1, 2, 3, 4, 6, 12}~~

(i)



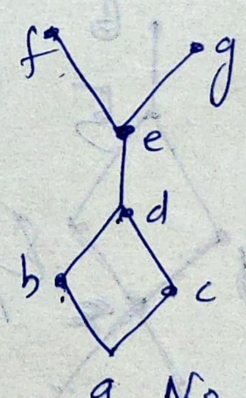
Yes

(ii)



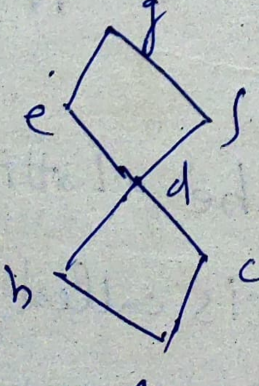
Yes

(iii)



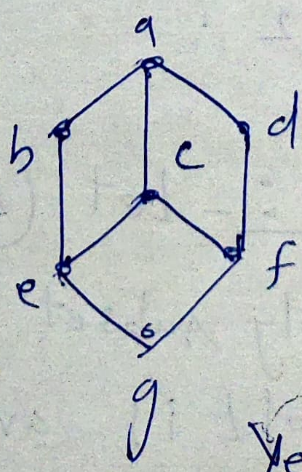
No

(iv)



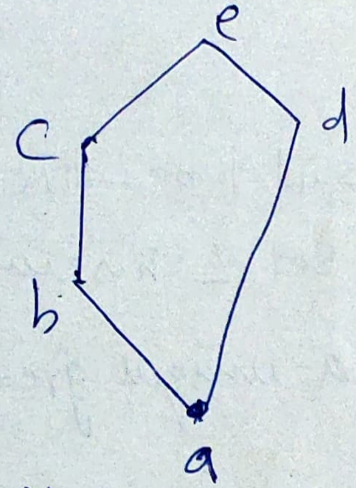
Yes

(v)



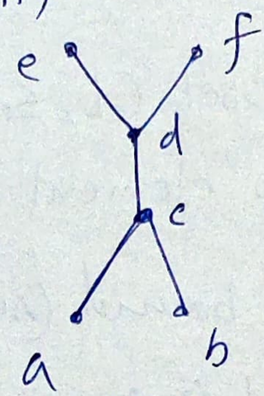
Yes

(vi)



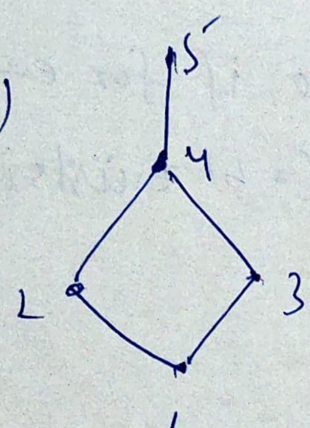
Yes

(vii)



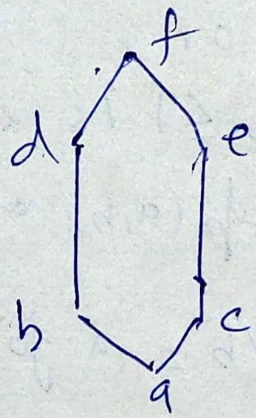
No

(viii)



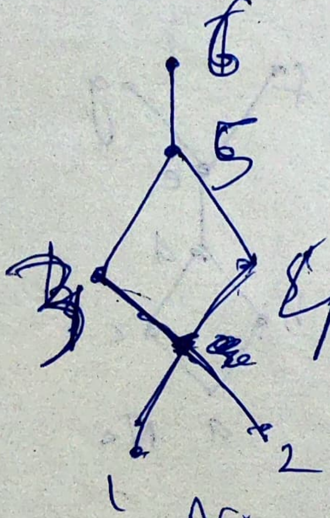
Yes

(ix)



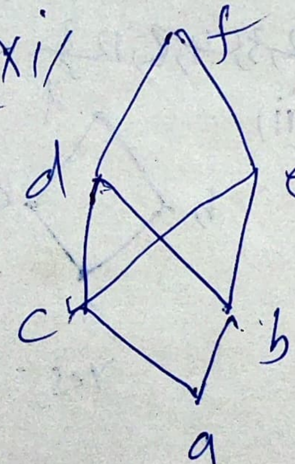
Yes

(x)



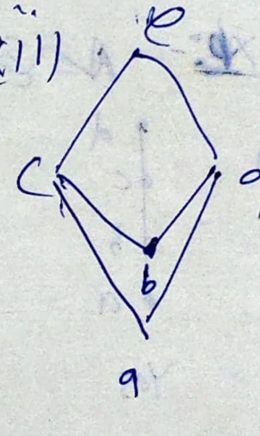
No

(xi)



No

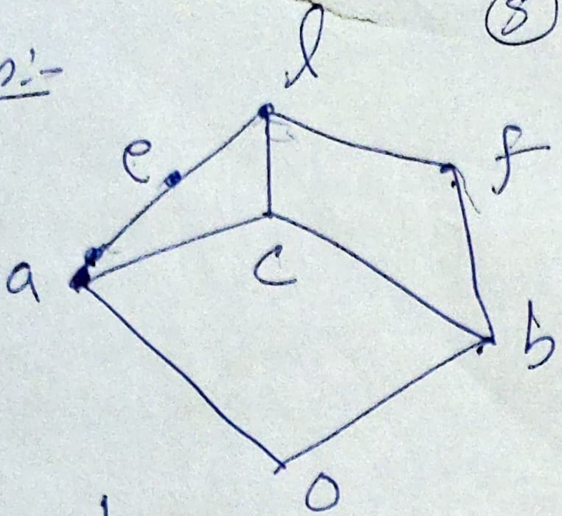
(xii)



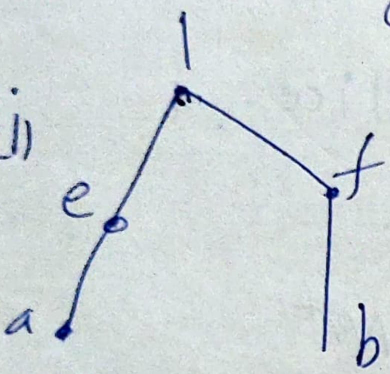
No

Sub Lattice :- Let $(L, \leq)$ be a Lattice,
A Non empty subset S of L is called a SubLattice of L if $a \vee b \in S$ and $a \wedge b \in S$ whenever $a, b \in S$

Exp:-



(i)

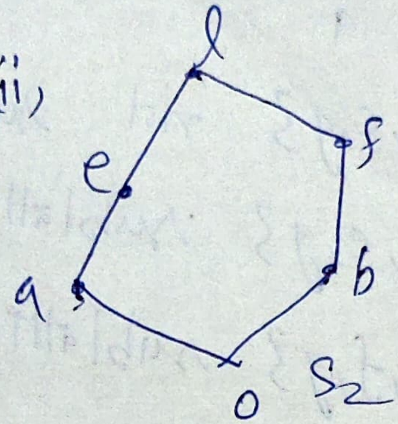


S_1

No

$a \vee b \notin S_1 \quad a \wedge b \notin S_1$

(ii)

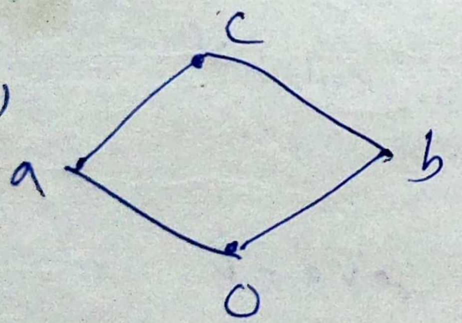


S_2

No becа

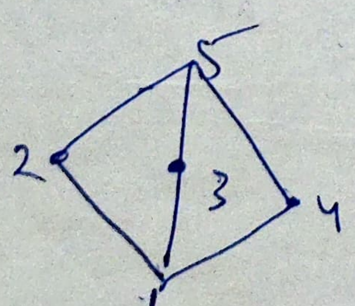
$a \wedge b \notin S_2$

(iii)



Yes

Exp:-

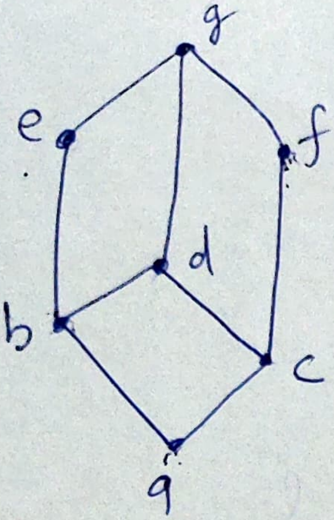


sublattice with Three or more elements

$\{1,2,5\}$ $\{1,3,5\}$ $\{1,4,5\}$ $\{1,2,3,5\}$ $\{1,3,4,5\}$

$\{1,2,4,5\}$, $\{1,2,3,4,5\}$ -

Exp:-



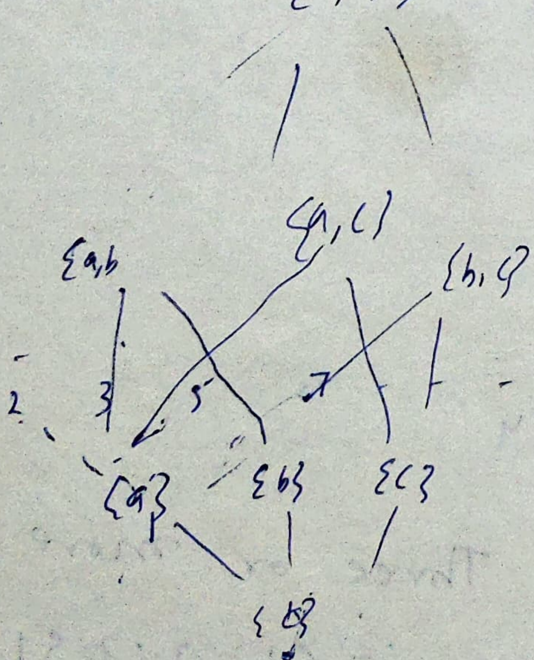
$L_1 = \{a, b, c, g\}$ not sublattice.

$L_2 = \{a, b, e, g\}$ sublattice.

$L_3 = \{b, d, f, g\}$ sublattice Not

~~$L_4 = \{a, d, f, g\}$ ni~~

$L_4 = \{a, d, f, g\}$ not subLattice.



greatest, last, unit

$a \vee a' = I$

$a \wedge a' = 0$

first, zero, least

Bounded Lattice:

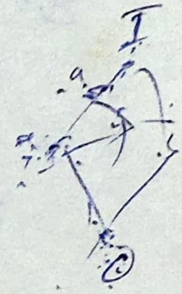
A Lattice L is said to be a bounded lattice if it has a greatest element I and a least element 0.

If L is a bounded Lattice then for any element $a \in L$, we have the following identities:

(i) $0 \le a \le I$

(ii) $a \vee 0 = a$, $a \wedge 0 = 0$

(iii) $a \vee I = I$, $a \wedge I = a$



*- Every finite Lattice is bounded.

Exp:-

① Z^+ under the partial order of divisibility is not bounded because it has least element 1, but no greatest element.

② $S = \{a, b, c\}$, $(P(S), \subseteq)$ is bounded lattice.

complement:

In a bound lattice $(L, \wedge, \vee, 0, I)$. An element $b \in L$ is called a complement of an element $a \in L$ if

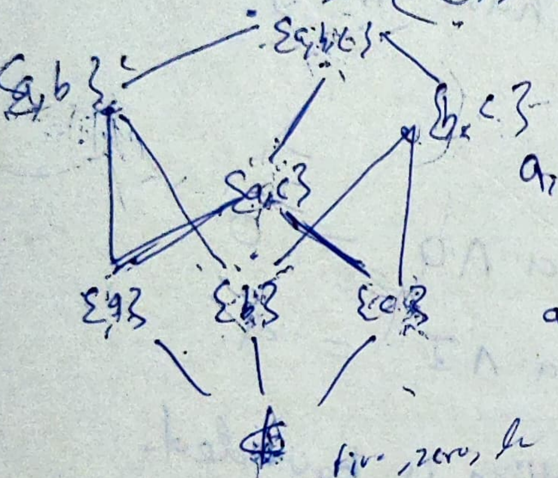
$a \vee b = I$ and $a \wedge b = 0$

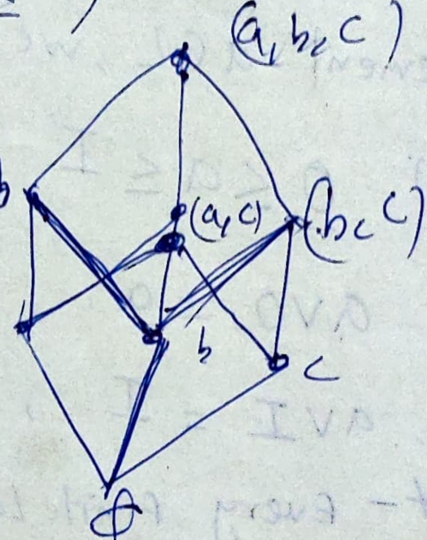
where I is greatest element & 0 is least element.

Complemented Lattice:

A Lattice L is called a complement lattice if it is bounded and every element in it has a complement.

Exp:- $S = \{a, b, c\}$, $(P(S), \subseteq)$ Yes





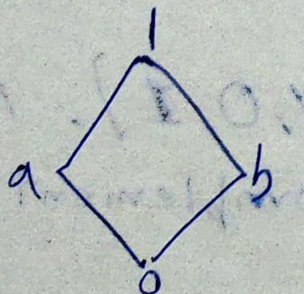
comp $a' = (b, c)$ & comp of $(b, c) = a$

comp $b' = (a, c)$ & comp of $(a, c) = b$

Comp $(c) = (a, b)$ comp $(a, b) = c$

Comp $\phi = (a, b, c)$ = c of $(a, b, c) = \phi$

Exp:-

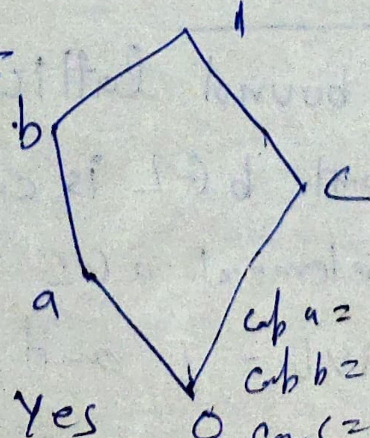


comp $(a) = b$ & comp $b = a$

com $(0) = 1$ & com $1 = 0$

Yes

Exp:-



Yes

comp a = [illegible]

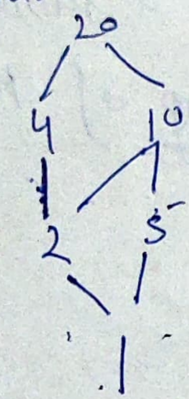
comp b = c

com c = a, b

com (0) = 1

⑦

Exp:- ③ consider the lattice D_{20} under the partial order of divisibility.



comp (1) = 20 & comp (20) = 1

comp (5) = (4) & comp (4) = 5

comp (2) = Nil

comp (10) = Nil

Not complemented lattice

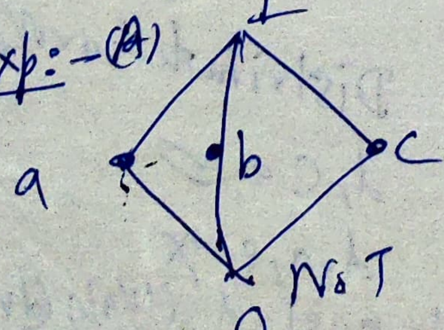
Distributed Lattice :-

A Lattice L is said to be distributed if for ~~any~~ (all) elements a, b, c in L. We have the following -

(i) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

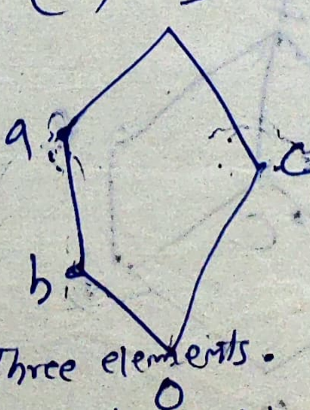
(ii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Exp:- (A)



NoT

(B)



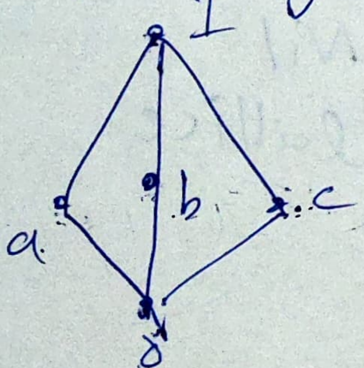
Not

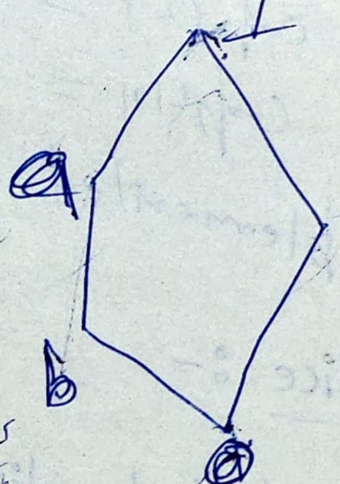
$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

* - for all combination of Three elements.
* - Every chain is distributed.

~~The~~ A Lattice is non distributed if and only if it contains a sublattice that is isomorphic to one of the following two lattice.

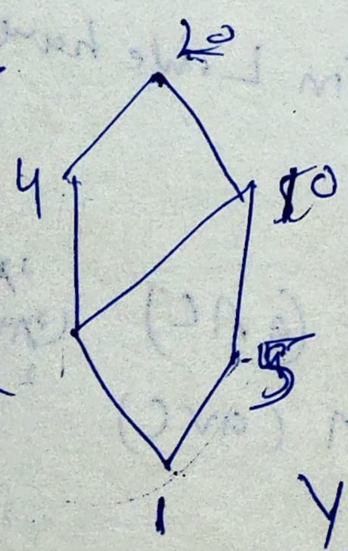




$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Exp:- D_{20} is Distributed or not.



✓ $2 \vee (4 \wedge 5) = 2 = (2 \vee 4) \wedge (2 \vee 5) = 2$

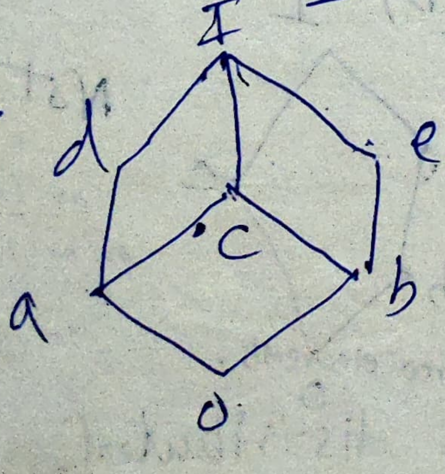
✓ $2 \wedge (4 \vee 5) = 2, \ (2 \wedge 4) \vee (2 \wedge 5) = 2$

✓ $4 \vee (5 \wedge 10) = 20, \ (4 \vee 5) \wedge (4 \vee 10) = 20$

✓ $4 \wedge (5 \vee 10) = 2, \ (4 \wedge 5) \vee (4 \wedge 10) = 2$

Yes

Exp:- Is Distributed or Not



d, c, a ✓

d, a, e ✗

$d \vee (a \wedge e) = d, \ (d \vee a) \wedge (d \vee e)\ d$

$d \wedge (a \vee e) = d, \ (d \wedge a) \vee (d \wedge e) = a$ ✗

Not

Complete lattice :

Let $(L, \leq)$ be lattice. Then L is said to be complete if every subset A of L has gretest lower bound $(\wedge A)$ and Least upper bound $(\vee A)$.

Every complete lattice has least & greatest element.

~~Every finite lattice~~

Exp:- $(Z^+, \leq)$ not complete because Z^+ is also subset of itself & $\vee A$ donot exist, $\wedge A$ exist.

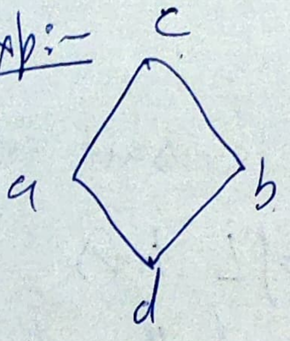
Modular lattice :

A lattice L is said to be modular lattice if for all $a, b, c \in L$, $a \leq c$

$$a \vee (b \wedge c) = (a \vee b) \wedge c$$

Every distributed lattice is modular

Exp:-



$a \vee (b \wedge c) = (a \vee b) \wedge c$ [a, b, c] $a \le c$

$c = c$

$b \vee (d \wedge c) = (b \vee d) \wedge c$ [b, d, c] $b \le c$

$b = b$

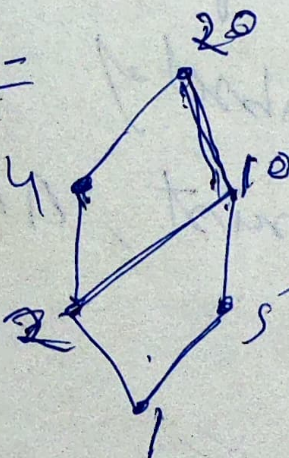
$d \vee (a \wedge b) = (d \vee a) \wedge b$ [d, a, b] $d \le b$

$d = d$

Hence modular lattice.

$^5C_3 = \frac{\lfloor 5}{\lfloor 3 \lfloor 2}$ $5 \times 2 = 10$

Exp:-



[2, 5, 4]

$2 \vee (5 \wedge 4) = (2 \vee 5) \wedge 4$

$2 = 2$

[5, 4, 10]
a b c

$5 \vee (4 \wedge 10) = (5 \vee 4) \wedge 10$

$10 = 10$

Hence it [4, 5, 20]

~~$(4 \vee 5) \wedge 20$~~

$4 \vee (5 \wedge 20) = (4 \vee 5) \wedge 20$

$20 = 20$

Hence modula

Morphism of Lattice:

Let L and M be lattice. A mapping $f: L \to M$ is called,

(i) Join-homomorphism if $f(x \vee y) = f(x) \vee f(y)$

(ii) Meet-homomorphism if $f(x \wedge y) = f(x) \wedge f(y)$

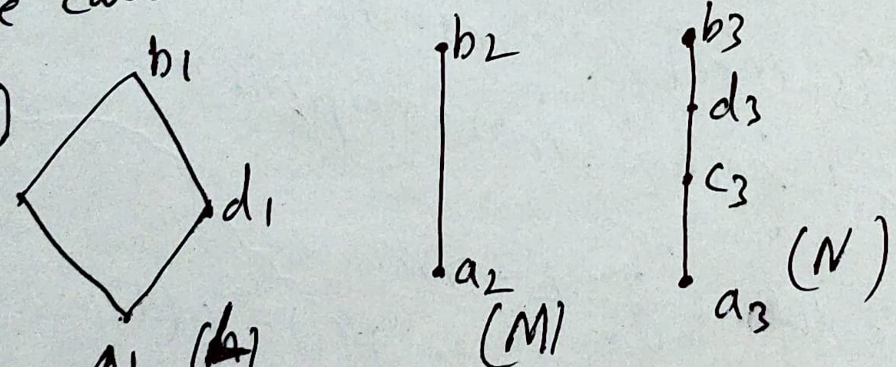
(iii) order-homomorphism if $x \leq y \Rightarrow f(x) \leq f(y)$:

for all $x, y \in L$.

The mapping f is called homomorphism (or lattice homomorphism) if f is both join and meet homomorphism.

If a homomorphism f is bijective (one to one & onto) then f is called isomorphism and L and M are called isomorphic lattice.

Exp: 1



(i) If we define $f: L \to M$.

$f(a_1) = f(c_1) = f(d_1) = a_2$, $f(b_1) = b_2$

$f(a_1 \vee b_1) = f(b_1) = b_2$

$f(a_1) \vee f(b_1) = a_2 \vee b_2 = b_2$ } True

$f(c_1 \vee d_1) = f(b_1) = b_2$ & $f(c_1) \vee f(d_1) = a_2 \vee a_2 = a_2$ false

not join Homo

$f(a_1 \wedge b_1) = f(a_1) = a_2$
$f(a_1) \wedge f(b_1) = a_2 \wedge b_2$
$= a_2$

True

$f(c_1 \wedge b_1) = f(c_1) = a_2$
$f(c_1) \wedge f(b_1) = a_2 \wedge b_2$
$= a_2$

True

$f(a_1 \wedge c_1) = f(c_1) = a_2$
$f(a_1) \wedge f(c_1) = a_2 \wedge a_2 = a_2$

True.

$f(c_1 \wedge d_1) = f(a_1) = a_2$
$f(c_1) \wedge f(d_1) = a_2 \wedge b_2$
$= a_2$

Ture

so f is meet & order homomorphism.
but not join & homomorphism.

(ii) If we define $g : L \to M$
$g(a_1) = a_2$; $g(b_1) = g(c_1) = g(d_1) = b_2$

$g(a_1 \vee c_1) = g(c_1) = b_2$
$g(a_1) \vee g(c_1) = a_2 \vee b_2 = b_2$

True

$g(a_1 \vee b_1) = g(b_1) = b_2$
$g(a_1) \vee g(b_1) = a_2 \vee b_2 = b_2$

True

$g(c_1 \vee d_1) = g(b_1) = b_2$
$g(c_1) \vee g(d_1) = b_2 \vee b_2 = b_2$ True

$g(c_1 \vee b_1) = g(b_1) = b_2$
$g(c_1) \vee g(b_1) = b_2 \vee b_2 = b_2$ True

So it is join homomorphism.

$g(a_1 \wedge c_1) = g(a_1) = a_2$
$g(a_1) \wedge g(c_1) = a_2 \wedge b_2 = a_2$ True

$g(a_1 \wedge b_1) = g(a_1) = a_2$
$g(a_1) \wedge g(b_1) = a_2 \wedge b_2 = a_2$ True

$g(c_1 \wedge d_1) = g(a_1) = a_2$
$g(c_1) \wedge g(d_1) = b_2 \wedge b_2 = b_2$ false

So g is not meet & Homomorphism.

(iii) If we define $h : L \to N$
$h(a_1) = a_3,\ h(c_1) = c_3,\ h(d_1) = d_3,\ h(b_1) = b_3$

$h(c_1 \vee d_1) = h(b_1) = b_3$
$h(c_1) \vee h(d_1) = c_3 \vee d_3 = d_3$ False

So not join Homomorphism

$h(c_1 \wedge d_1) = h(a_1) = a_3$

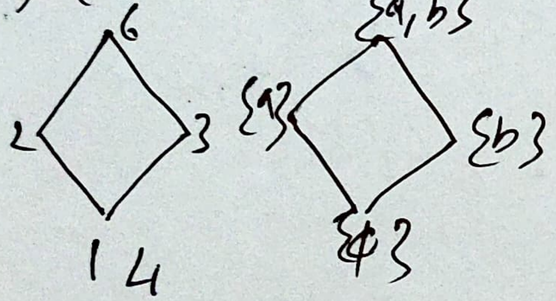
$h(c_1) \wedge h(d_1) = c_3 \wedge d_3 = c_3$ } false

so not meet homomorphism.

Exp: (2) L_1 Divisor of 6 under divisibility and $S = \{a, b\}$, $P(S), \subseteq$ is L_2.

$f : L_1 \to L_2$ is defined

$f(1) = \phi$, $f(2) = a$, $f(3) = b$, $f(6) = \{a, b\}$



$f(2 \vee 3) = f(6) = \{a, b\}$

$f(2) \vee f(3) = \{a\} \vee \{b\} = \{a, b\}$ } True

$f(1 \vee 2) = f(2) = \{a\}$

$f(1) \vee f(2) = \{\phi\} \vee \{a\} = \{a\}$ } True

$f(2 \vee 6) = f(6) = \{a, b\}$

$f(2) \vee f(6) = a \vee b = \{a, b\}$ } True

so it is join homomorphism.

$f(2 \wedge 3) = f(1) = \phi$

$f(2) \wedge f(3) = \{a\} \wedge \{b\} = \phi$ } True

$f(1 \wedge 2) = f(1) = \phi$

$f(1) \wedge f(2) = \phi \wedge a = \phi$ } True

$f(2 \wedge 6) = f(2) = a$

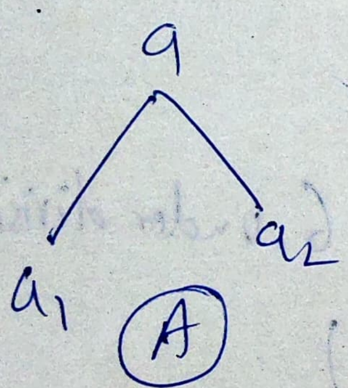
$f(2) \wedge f(6) = \{a\} \wedge \{a, b\} = a$ } True

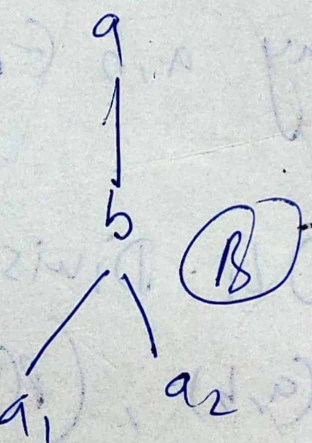
so f is meet homomorphism. f is bijective and join & meet so f is isomorphism and L_1 & L_2 are isomorphic.

Join-Irreducible element :-

Let L be a lattice with least element 0. An element $a \neq 0$ in L is said to be join irreducible if $a = a_1 \vee a_2$ implies $a = a_1$ or $a = a_2$.

- Also a is a join irreducible iff a has a unique predecessor.





ans:- A

(A) a is not join irreducible element

(i) because - $a_1 \vee_2 = a$ but $a_1 \neq a_2$

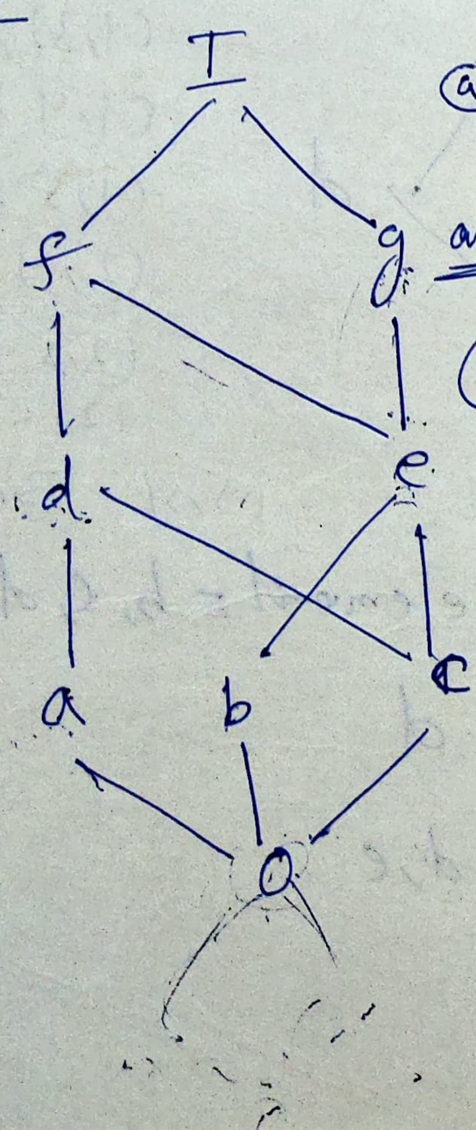
(ii) because - a has not unique predecessor

Atom:- Let L be a lattice with least element 0. Those elements which have 0 unique predecessor are called atom.

Meet - Irreducible element:

Let L be a lattice with greatest element I. An element a in the lattice is said to be meet irreducible if $a = a_1 \wedge a_2$ implies $a = a_1$ or $a = a_2$ and iff a has only one immediate successor.

Exp:-



(a) Join-irreducible element

ans → a, b, c, g

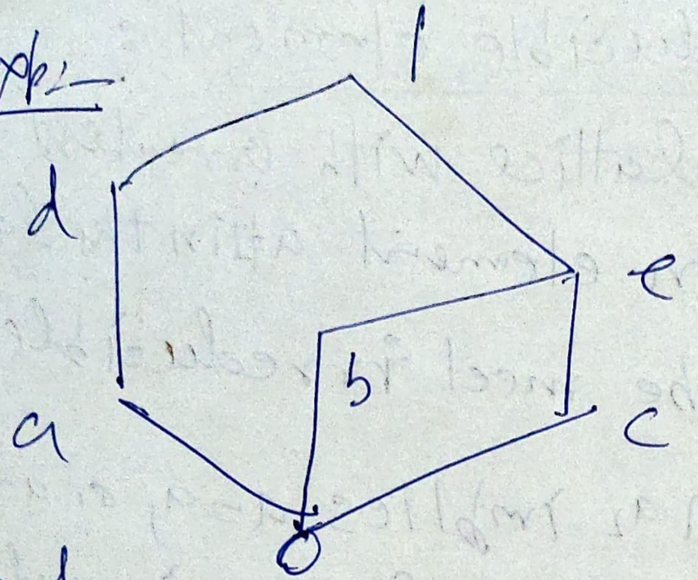
(b) Find atoms

a, b, c

(c) Find meet-irreducible

a, b, d, f, g

Exp:-



find
① meet irreduciable t_2

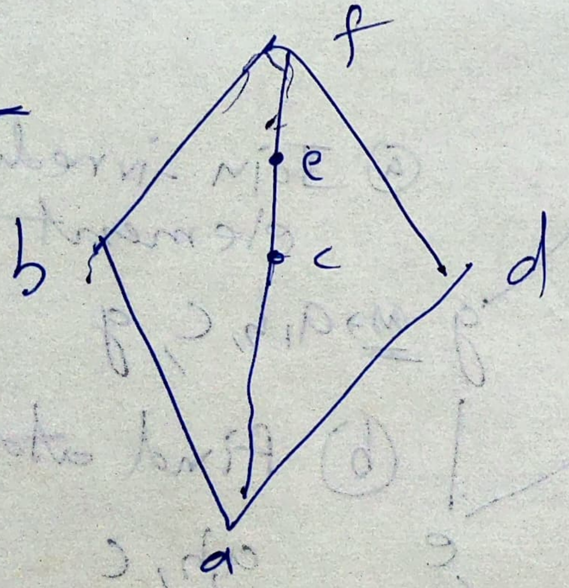
ans:- d, e, a, c, b

② atoms
ans:- a, b, c
③ Join
a, b, c, d

1, 2, 3, 4, 5

(1, 1)
(1, 2)
(1, 3)
(1, 4)
(1, 5)
(2, 4)
(2, 5)
(2, 2)
(3, 3)
(5, 5)
(4, 4)

Exp:-



Join irreducible element = b, c, d, e

atoms = b, c, d

meet = b, c, d, e

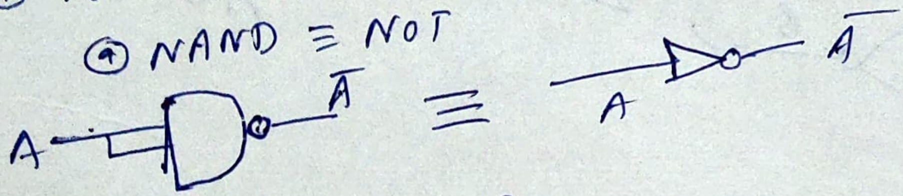
~~Properties of Lattice:~~

Gates & circuits:

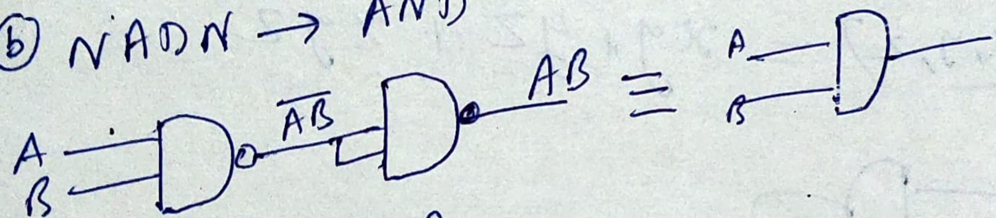
Universal Gates:

① NAND ② NOR

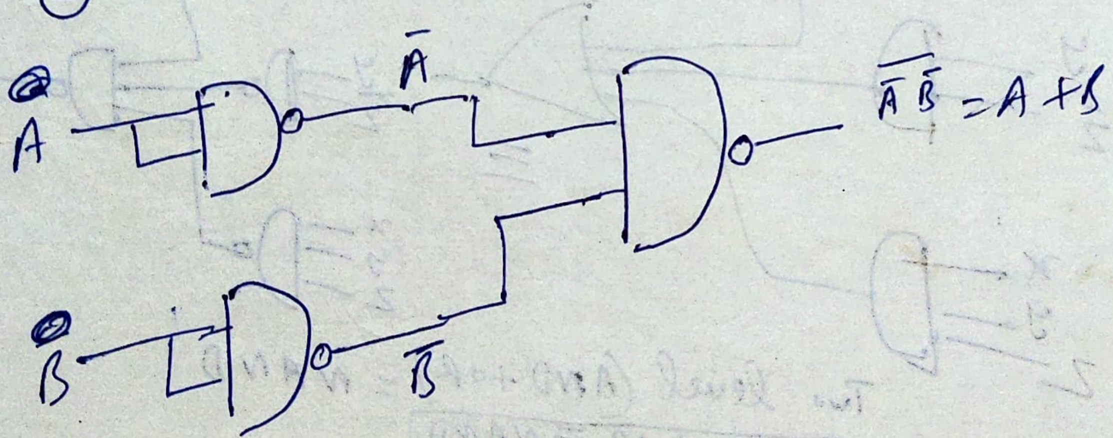
ⓐ NAND ≡ NOT



ⓑ NADN → AND

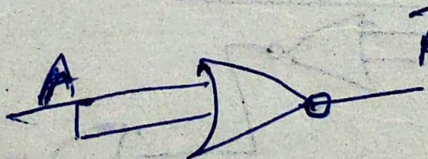


ⓒ NAND → OR

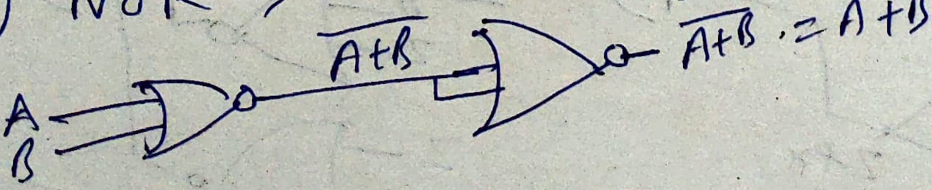


② NOR:-

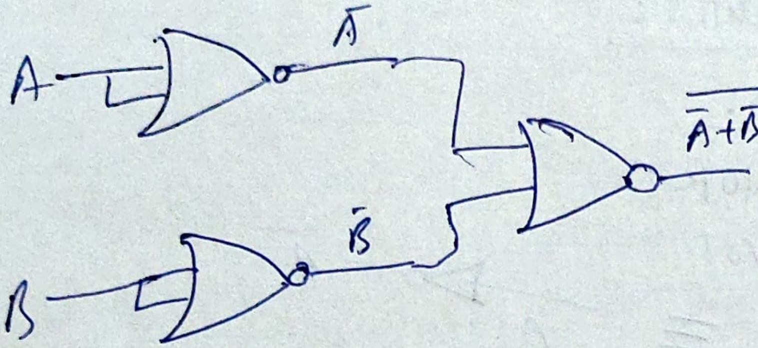
ⓐ NOR → NOT



ⓑ NOR → OR

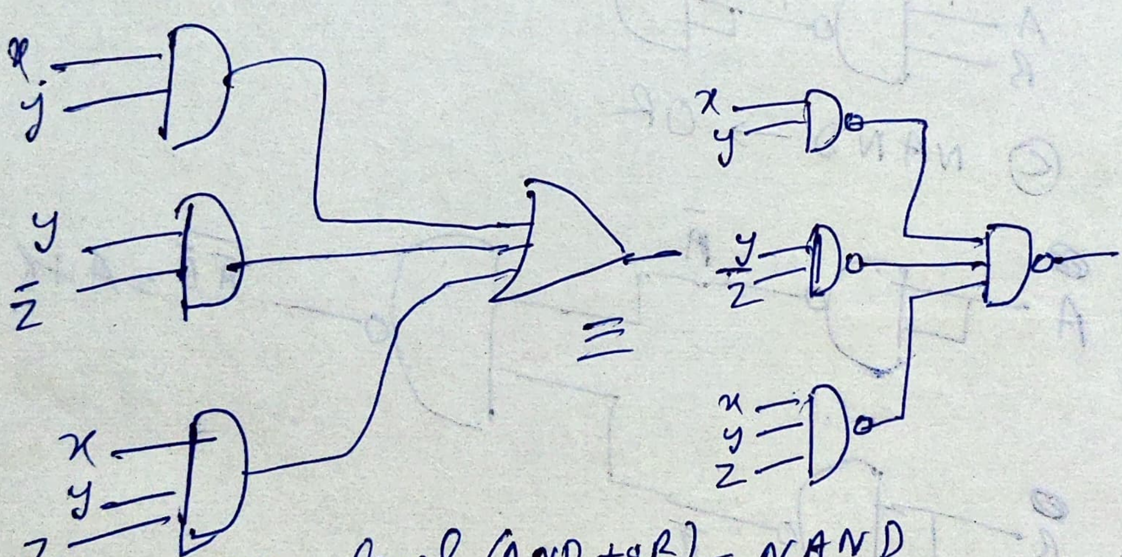


(c) NOR → AND



$= AB$

(1) $f(x,y,z) = xy + y\bar{z} + xyz$



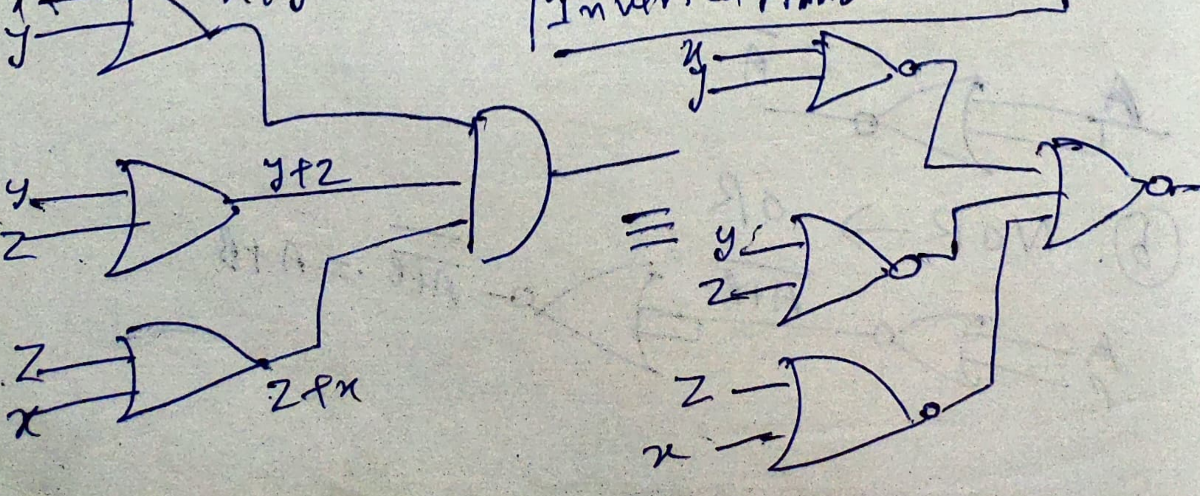
Two level (AND + OR) = NAND

$\boxed{\text{Inverted OR} \equiv \text{NAND}}$

(2) $f(x,y,z) = (x+y)(y+z)(z+x)$

Two level (OR + AND) = NOR

$\boxed{\text{Inverted AND} \equiv \text{NOR}}$



(12)

Draw the logic ckt :-

① $Y = AB'C + AC' + B'C$
also draw the ckt using only NAND gates.

② $Y = A'BC + A'BC' + ABC'$

Booleam Algebra :

Let B be a non empty set with two binary opern +, *, ' and two distinct elements 0, 1. Then B is called a boolean algebra if the following axioms hold ~~where~~ for every $a, b, c \in B$.

① Commutative law

ⓐ $a+b = b+a$ ⓑ $a*b = b*a$

② Identity law

ⓐ $a+0 = a$ ⓑ $a*1 = a$

③ Distributed law

ⓐ $a+(b*c) = (a+b)*(a+c)$ ⓑ $a*(b+c) = (a*b)+(a*c)$

④ complement law

ⓐ $a+a' = 1$ ⓑ $a*a' = 0$

Duality :- Dual of f^n can be obtain by interchangin [1 → 0 and 0 → 1] and [+ → · and · → +]

Exp:- ① $x\bar{x} = 0$
Dual of func
$x + \bar{x} = 1$

② $xy + \bar{y}z = 1$
Dual of fun
$(x+y)\cdot(\bar{y}+z) = 0$

Find complement :-

① **By Dual method :-**

ⓐ First take dual of function

ⓑ Then complement each variable

② **By Demorgan's* law :-**

$(a+b)' = a' \cdot b'$

$(a \cdot b)' = a' + b'$

Exp :- $xy + \bar{y}z$

① By Dual method

Dual of $xy + \bar{y}z$

$= (x+y) \cdot (\bar{y}+z)$

complement of each variable

$= (\bar{x}+\bar{y}) \cdot (y+\bar{z})$

② By Demorgan's law -

$(xy+\bar{y}z)' = \overline{xy} \cdot \overline{\bar{y}z}$

$= (\bar{x}+\bar{y}) \cdot (y+\bar{z})$

Properties of boolean algebra :-

1. $x \cdot \bar{x} = 0 \quad , \quad x + \bar{x} = 1$ Identity

2. $(x+y)+z = x+(y+z)$
   $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ Associative

3. $x+y = y+x \quad , \quad x \cdot y = y \cdot x$ Commutative

4. $x(y+z) = x \cdot y + x \cdot z$
   $x+(y \cdot z) = (x+y) \cdot (x+z)$ Distributed

5. $\cancel{x \neq 0}$ $\overline{x+y+z+\cdots} = \bar{x} \cdot \bar{y} \cdot \bar{z} \cdots$
   $\overline{x \cdot y \cdot z \cdots} = \bar{x} + \bar{y} + \bar{z} + \cdots$ Demorgan's law

6. $A + \bar{A}B = A + B \quad , \quad A + AB = A$

Literal:- It is a variable or complement of a variable.

Fundamental product: Fundamental product is a literal or product of two or more literals in which no two literal involve the same variable.

Exp:- xz', $xy'z$, x, y', $x'yz$ are fundamental product.

but $xyzx$, $xyzx'$ are not

Sum of product expression: A boolean expression E is called a SOP expression if E is fundamental product or the sum of two or more fundamental products none of which is contained in another.

Exp:- $E_1 = xz' + y'z + xyz'$.

$E_2 = xz' + x'yz' + xy'z$

E1 is not SOP expression because xyz' contains xz'.

E2 is a SOP expression

P.T.O

(Disjunctive Normal Form or Disjunctive canonical form):

Complete Sum of Product form :

A boolean expression $E = E(x_1, x_2, \text{---} x_n)$ is said to be a complete SOP expression if E is SOP expression where each product P involves all the n variables. Such a fundamental product which involves all the variable is called a Minterm.

$\Sigma(m) =$

Exp:-

$x \rightarrow 1$

$\bar{x} \rightarrow 0$

$E = x(y'z)'$

Find SOP & complete SOP.

$E = x(y'z)'$

$E = x(y + z')$

$E = xy + xz'$

which is SOP.

$E = xy(z+z') + xz'(y+y')$

$= xyz + xyz' + xyz' + xy'z'$

$= xyz + xyz' + xy'z'$

$= m_7 \quad m_6 \quad m_4$

$= \Sigma m(4, 6, 7)$

$\Rightarrow xy + \cancel{xz'}$

$xy + xy'z'$

$x[y + y'z']$

$x[(y+y') \cdot yz']$

$x \quad y \quad z'$

K

K-map:

K-map is a graphical method of minimizing any boolean function just by observation.

It is nothing a rectange made up of certain number of squares each square representing minterm or Maxterm.

Two variable:

a \ b	0	1
0	m_0	m_1
1	m_2	m_3

Three variable:

a \ bc	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Four Variable:

ab \ cd	00	01	11	10
00	0	2	3	1
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Five Variable:

$\bar{A}$

BC \ DE	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

A

BC \ DE	00	01	11	10
00	16	17	17	18
01	20	21	23	22
11	28	29	31	30
10	24	25	27	26

6-variable :

CD\EF, $\bar{A}\bar{B}$

CD\EF	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

CD\EF, $\bar{A}B$

CD\EF	00	01	11	10
00	16	17	19	18
01	20	21	23	22
11	28	29	31	30
10	24	25	27	26

CD\EF, $A\bar{B}$

CD\EF	00	01	11	10
00	32	33	35	34
01	36	37	39	38
11	44	45	47	46
10	40	41	43	42

CD\EF, AB

CD\EF	00	01	11	10
00	48	49	51	50
01	52	53	55	54
11	60	61	63	62
10	56	57	59	58

Don't care :-

funct^n not specified for some values.

always $1 \to d$.

Note:- $f(d) = \Sigma(2,3,12,13,14,15) \rightarrow 1$

$= \Pi(0,1,4,5,6,7,8,9,10,11)$ $\downarrow 0$

0	1	3d	2d
4	5	7	6
d12	13d	15d	14d
8	9	11	10

Exp:- find SOP & complete SOP.

$E = z(x'+y) + y'$

Exp:- $E = xy' + xyz' + x'yz'$

Prove that

(a) $xz' + E = E$ (b) $x + E \neq E$

Complete Disjunctive Normal forms:-

A DNF in n variables which contains all the possible 2^n terms is called complete Disjunctive Normal form.

Exp:- complete DNF in two variable

$xy + x'y + xy' + x'y'$.

A complete DNF is identically 1.

Conjunctive Normal form Or complete Product of some Or conjunctive canonical form: A boolean expression is in CNF if it is a product of maxterm.

Complete conjunctive Normal form:-

A CNF in n variable which contains all the possible 2^n factors is called complete CNF.

Ex:- Complete CNF in two variable
$$(\bar{x}+\bar{y})(\bar{x}+y)(x+\bar{y})(x+y).$$

Maxterm:- $\wedge(M)$

A boolean expression in k variable $x_1, x_2 \ldots x_k$ is called maxterm if it is of the form $y_1 + y_2 + y_3 + \ldots y_k$ or sum of k ~~element~~ distinct variables (none of which involves the same variable), where y_i is a literal (either x_i or $\bar{x}_i$) for $1 \le i \le k$ $y_i \neq y_j$ for $i \neq j$.

$\bar{x} \longrightarrow 1$

$x \longrightarrow 0$

A complete CNF is identically 0

$f = \Sigma(1,3,5,6,7)$

$f(d) = (0,2,4,5,6)$

Exp:- 1

$f(\cancel{d}) = \pi M(0,2,4)$

$f(d) = \pi M(1,3,7)$

Exp:- 2 $f(A,B,C,D) = \Sigma(0,1,2,4,5,6,8,9,12,13,14\}$

ans:- $\bar{C} + \bar{A}\bar{D} + B\bar{D}$

also find Disjunctive Normal form.

Exp:- 3

$f(A,B,C,D,E) = \Sigma(0,2,4,6,9,11,13,15,17,21,25,27,29,31)$

ans:- $\bar{A}\bar{B}\bar{E} \cancel{+\ldots} + A\bar{D}E + \underbrace{(A+\bar{A})}_{1}BE$

Exp:- 4

$f(a,b,c,d) = \pi(2,3,4,6,12,13,14,15)$

$f(d) = \Sigma(2,3,12,13,14,15)$

ans- $(d+\bar{b})$

also find Conjunctive Normal form.

Q) $f(A,B,C,D) = A'B'C' + B'CD' + A'BCD' + AB'C'$

Ans:- $B'C' + A'CD' + B'D'$

Q) $f(A,B,C,D) = \pi(\cancel{3,7,\ldots}\ 0,2,8,10,13,15,6)$

AB \ CD	00	01	11	10
00	0			0
01				0
11		0	0	
10	0			0

$= (B+D)\cdot(\bar{A}+\bar{B}+\bar{D})(A+\bar{C}+D)$

Also find conjuctive Normal for

AB \ CDE	000	001	011	010	110	111	101	100
00	0 1	1	3	2 1	6 1	7 1	5	4 1
01	8	9 1	11 1	10	14	15 1	13 1	12
11	24	25 1	27 1	26	30	31 1	29 1	28
10	16	17 1	19	18	22	23	21 1	20

B $B\bar{C}E$.

Poonam

Anher
Mitey
Preet
Jyoti
Sudha

Prove:- ①

$[x'(x+y)]' + [y\cdot(y+x')]' + [y(y'+x)]' = 1$

L.H.S

$= [x'(x+y)]' + [y\cdot(y+x')]' + [y(y'+x)]'$

$= x + (x+y)' + y' + (y+x')' + y' + (y'+x)'$

$= x + x'y' + y' + y'x + y' + yx'$

$= x + y' + y'(1+x) + y' + x'$ $\quad A+\bar{A}B = A+B$

$= x + y' + x'$

$= 1 + y'$

$= 1$

②

$(xy'+yz)\cdot(xz+yz') = xz$

L.H.S.

$= (xy'+yz)\cdot(xz+yz')$

$= xy'z + xyz + 0 + 0$

$= xz(y+y')$

$= xz\cdot 1$

$= xz$

③ $(x+yz)\cdot(y'+x)\cdot(y'+z') = x(y'+z')$

L.H.S. $= (x+yz)(y'+x)(y'+z')$

$= \cancel{xy' + xy'z' + xy' + xz'}$

$= (xy' + x + yx)(y'+z')$

$= [xy' + x(1+y)](y'+z')$

$= x[\underbrace{y'+1}_{=1}] + y' + z' = x(y'+z')$